On Dating, Match-Making, Marriage Stability and…College Admission

Math with neither equations nor numbers

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Content

- The match-maker's dilemma
- Mutual preferences
- Stability of marriage
- Alternative marriage [polygamy, same gender,…]
- Fair disclosure, honesty / dishonesty
- Conclusion: Math can deal with them all!

The match-maker's dilemma

Boy Preference Table

Abe	Bill	Charlie
Amy	Beth	Carol
Beth	Carol	Amy
Carol	Amy	Beth

Girl Preference Table

Amy	Beth	Carol
Bill	Charlie	Abe
Abe	Bill	Bill
Charlie	Abe	Charlie

Matchmaker Mission

- Propose a match to every girl and boy taking into account their preferences

- Make sure all accept the proposal

$\Downarrow$

stability

Stability

Definition: Marriage table is stable as long as there is no pair of a girl and a boy who both prefer to marry each other discarding the matchmaker proposal.

Example: Dianna +Charles Pamela +Harry

Boy Preference Table

Abe	Bill	Charlie
Amy	Beth	Carol
Beth	Carol	Amy
Carol	Amy	Beth

Girl Preference Table

Amy	Beth	Carol
Bill	Charlie	Abe
Abe	Bill	Bill
Charlie	Abe	Charlie

Following First Priorities

Abe	Bill	Charlie
Amy	Beth	Carol

Amy	Beth	Carol
Bill	Charlie	Abe

Notes:

1. 2 stable solutions for the above preference lists
2. “Stable” does not imply “best”

Questions

- Does the highest preference always provide a stable solution?
- Does a stable solution always exist?
- In case there is more than one stable solution which one is better, in what sense and to whom?
- Is there an algorithm providing the stable solution(s)?

Girl's Preferences

Amy	Beth	Carol
Bill	Charlie	Abe
Abe	Bill	Bill
Charlie	Abe	Charlie

Amy	Beth	Carol
Bill	Bill	Abe

Existence Theorem

- Does any pair of boys' and girls' preference lists has a stable solution?

- Relevance:
  - Search for an algorithm
  - Generating a computer program

The Roommate Problem

Abe	Bill	Charlie	David
David	Abe	Abe	Charlie
Charlie	Charlie	David	Abe
Bill	David	Bill	Bill

(Bill ~ bad guy…)

Algorithm

- Often only following existence
- Occasionally serves as a proof for existence
- “Boy’s initiative” algorithm

  “Dating Days” / “Matching Month”

  - Always converges – when each girl accepted an offer at least once
  - Always stable!
  - Initiative pays! – Never be passive!
  - Girl’s initiative

- Demonstration
- Guarantees existence
- Can be easily implemented in a computer code

Alternative Marriage

- Polygamy
- Same gender matching
- Non-marriage matching:
  - College/University admission [US, GB]
  - Roommate assignment
  - Whose initiative? Whose advantage?
  - Doctors and lawyers assignment [spouse problem]
  - Auctions [but not stock exchange!]
  - Early universe cosmology

“Cheating”

- Disclosed vs. undisclosed lists
- Fair vs. unfair use; cheating

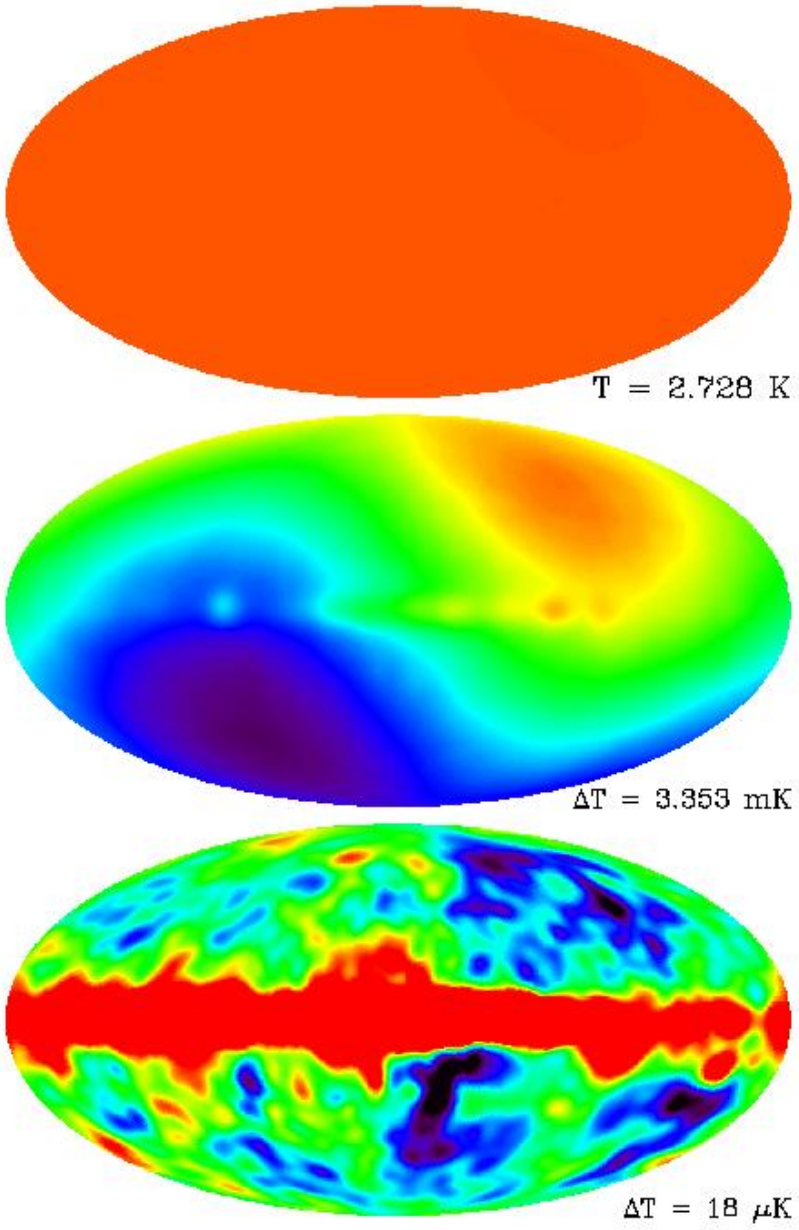
Amy	Beth
Abe	Bill
Bill	Abe
Charlie	…

Summary

- Stable two sided matching is always possible
- Single sided – may not be possible
- Girls too should initiate!
- Being passive leads to the worst possible match!
- Polygamy – ok!
- Spouse problem of lawyers and MDs
- Math can handle all problems [cheating included]

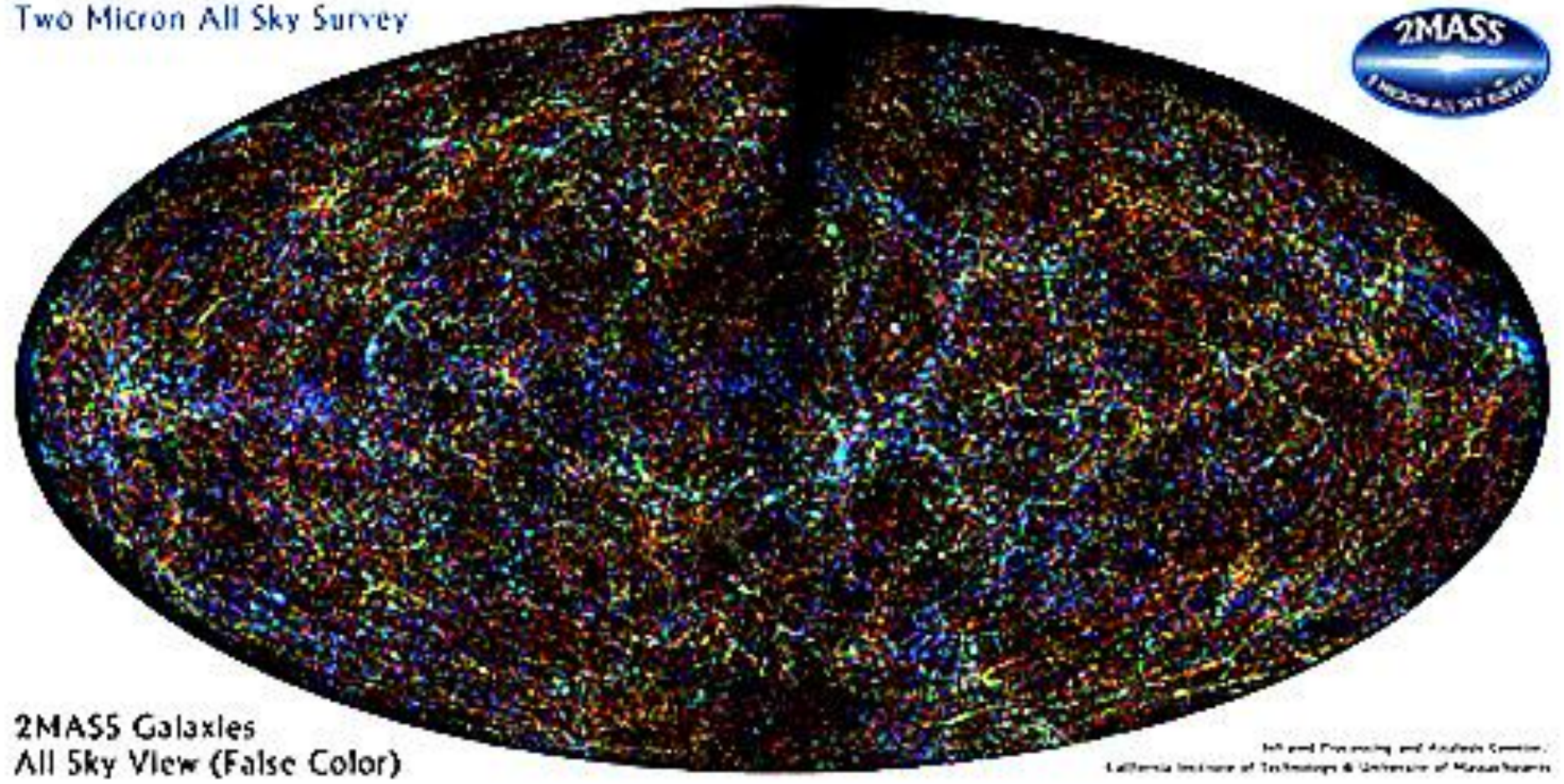
Galaxy Formation

Microwave Background Temperature distribution



Galaxy Formation

Galaxy Distribution Map



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